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PREDICTIVE METHODS FOR TWO-PHASE FLOW PRESSURE LOSS IN TEE JUNCTIONS WITH COMBINING CONDUITS

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Abstract—This paper discusses several predictive methods for two-phase flow pressure loss calculation in tee junctions with combining conduits. A semi-empirical approach has been followed utilizing three different models for the prediction of the pressure drop at the junction. The models are referred to as "loss coefficient model", "contraction coefficient model" and "momentum coefficient model". The results were compared with experimental data that were measured in a refrigerant flow loop.

Key Words: tee junctions, pressure drop, two-phase flow

1. INTRODUCTION

A typical design problem in nuclear and chemical industry is evaluating the two-phase pressure change in branching conduits. For many elements of these conduits (e.g. sudden enlargements, sudden contractions, separating tee junctions or bends), models have been developed to determine the two-phase pressure drop (gas/liquid). However, little information is given in the literature for tee junctions with combining conduits. To the best knowledge of the authors, only Mayinger (1982) has suggested evaluating the pressure drop via a homogeneous procedure, which is analogous to the procedure for single-phase conditions. Although there exists only little information on tee junctions with combining conduits the way is very much predetermined concerning how to set up a method for the evaluation of the pressure drop. A purely analytical approach is not feasible, because the knowledge on two-phase flow is not sufficient. On the other hand an empirical approach would involve too many independent parameters. This is the reason for pursuing a semi-empirical approach using three different models for the prediciton of the pressure drop in tee junctions. Pressure drop measurements for various combinations of two-phase flow pattern were made and each model tested by comparison with the experimental data. The models are derived from known models, which can be classified as follows:

(1) *Models exclusively based on the mechanical energy balance:* in these models the pressure change is divided into a reversible component (estimated by the mechanical energy balance) and an irreversible component [estimated as the product of a loss coefficient and the kinetic energy flux (e.g. Saba & Lahey 1984) for the branching conduit of a separating tee junction].
(2) *Models using flow-contraction coefficients:* in these models the flow through an element of a conduit is split into two parts. The pressure change up to the maximum flow-contraction is estimated with the mechanical energy equation and the pressure change after that with the momentum equation (e.g. Seeger 1985; for separating tee junctions).
(3) *Models using only the momentum equation:* in these models the pressure change is estimated by a simplified momentum balance, which is corrected by a coefficient (e.g. Saba & Lahey 1984; for the running conduit of a separating tee junction).

The developed models contain specific parameters, here termed "open parameters", which had to be determined. They were chosen based on physical and/or methodological assumptions, while some of these assumptions were also based on visual observations made at an air–water loop with a transparent test section. The caluclated values for the pressure changes were then compared to those measured at a refrigerant test loop.

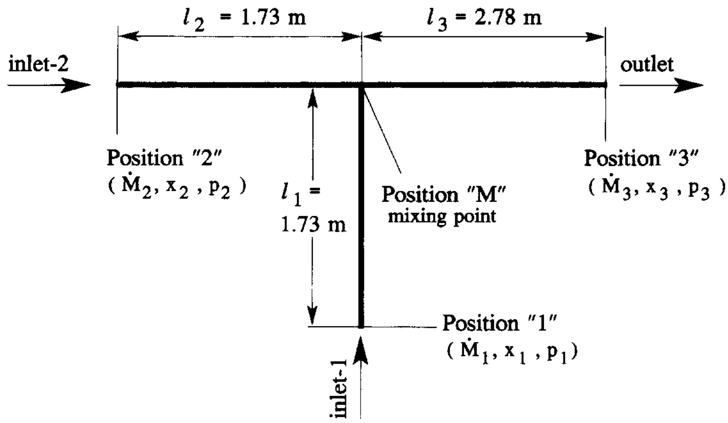


Figure 1. Tee junction with combining conduits. Pressure drop measurements for the experiments with the refrigerant test loop were made along the given pipe length for different inlet mass flow rates ($\dot{M}_1$, $\dot{M}_2$), inlet qualities (x_1, x_2) and pressures (p_1, p_2).

1.1. Definition of the Problem

Figure 1 shows a conduit with a combining tee junction and some notations used in this paper. For the design purpose, the estimate of the total pressure drop between inlet flow position "1" or "2" and outlet "3" is important. It is postulated that the flow patterns at these three locations are similar to an undisturbed pipe flow under equivalent flow conditions. It is further defined that the total pressure drop $\Delta p_{i\text{-}3}^{(t)}$ (where suffix "i–3" denotes the run from position "i" to "3") consists of two terms:

$$\Delta p_{i\text{-}3}^{(t)} = \Delta p_{i\text{-}3}^{(T)} + \Delta p_{i\text{-}3}^{(p)}; \quad i = 1 \text{ or } 2 \qquad [1]$$

Term $\Delta p_{i\text{-}3}^{(p)}$ stands for the sum of the pressure drops for undisturbed pipe flows, across the inlet and outlet branches of the tee, i.e. in case there is no phase change and the fluid parameters are constant then

$$\Delta p_{i\text{-}3}^{(p)} = \Delta p_{i\text{-}M}^{(p)} + \Delta p_{M\text{-}3}^{(p)} = \left(\frac{\Delta p}{\Delta z}\right)_i l_i + \left(\frac{\Delta p}{\Delta z}\right)_3 l_3, \qquad [2]$$

where $(\Delta p/\Delta z)_i$ stands for the pressure gradient in leg "i".

Term $\Delta p_{i\text{-}3}^{(T)}$ is the part of the pressure change that should cover the influence of the tee junction and is the subject of this investigation. Term $\Delta p_{i\text{-}3}^{(t)}$ will be measured.

2. EXPERIMENTS

Qualitative experiments were first carried out in an air–water loop (figure 2) for visual observation of the gas–liquid flow behaviour around the tee junction. All three inner diameters of the tee branches were equal ($d = 20$ mm). The volumetric flow rates (Q) were varied in the experiments between 0 and 1 m^3/h for liquid and 18 m^3/h for air. The main flow direction was horizontal in all experiments, while the orientation of flow-1 of the side branch was changed from upwards to horizontal and to downwards. Pictures with a high-speed video camera were performed to provide enhanced visualization of the flow at the junction.

In case there was only liquid flow through one inlet branch the flow mixing behaviour could well be observed. Two examples for combining a slug flow with a liquid flow are shown in figures 3 and 4. These pictures show clearly, that the flow is contracted—at least the gas flow. Similar flow contraction behaviour is known for single-phase flow (e.g. measured by Schmid 1977).

In many experiments (see Schmidt 1993) with flow through only one inlet and no flow through the other inlet, a certain reverse flow into the pipe filled with stagnant water could be identified.

At high gas and liquid velocities for both flows to the tee junction no identification of the flow regimes was possible; but there was nevertheless a strong indication that mixing of flow-1 and -2 occurs directly after the tee junction.

Quantitative experiments were carried out at a refrigerant test loop, which is schematically shown in figure 5. The pipe diameters of the tee legs were equal in all experiments ($D = 27.3$ mm). The tee junction itself was drilled out of a compact piece of material. Inlet-2 flow was horizontal while inlet-1 flow was vertical upwards. The two mass-flow rates to each inlet of the tee were adjusted separately. The liquid mass flows have been measured after the preheater and the gas mass flows after the superheater, which is a directly electrical-heated pipe after the evaporator.

The pressure gradients along the legs of the tee were measured by scanning neighbouring measuring taps. The pressure tappings were designed as toroidal chambers around the tube walls, connected to the inside of the tube by eight 1.2 mm equally spaced holes at an angle of 67.5° in the flow direction. Figure 6 shows typical pressure gradients for single- and two-phase flow. The curves look very similar. Extrapolating the curves backwards from their linear part at the outlet branch to the geometrical mixing point "M" of the junction, results in the pressure drops assignable to undisturbed flows. The pressure curves drop steeply just after the tee junction and pass through local minima indicating a deadwater area. After these local minima the shapes of the pressure gradients differ for single- and two-phase flow. The pressure distribution for single-phase flow is linear, while for two-phase flow it is non-linear. A reasonable explanation for this typical two-phase effect is the homogenization of flow around the deadwater area. Further downstream the flow pattern approaches the fully developed pipe flow.

The effect of the two-phase flow on the pressure drop will be considered by term $\Delta p_{\mathrm{ad}}^{(\mathrm{T})}$ that shall be further called "additional pressure change". Measurements were made at the following reduced pressures: $p/p_c = 0.2, 0.375, 0.5$ and 0.75 ($p_c = 4.13$ MPa, critical pressure of the fluid R12). At each pressure level the fluid parameters were fixed and for a given geometry the pressure drops $\Delta p_{i-3}^{(\mathrm{T})}$ are only a function of the known two inlet mass flow rates. Each flow was known by measuring the saturated water flow rate and the vapour flow rate separately before mixing. These four independent parameters can be reduced only to three via the dimensional analysis. The following three mass flow ratios, the values of which lay between zero and one, are suitable to characterize the range of the performed experiments, shown in figure 7.

$$\Pi_1 = \frac{\dot{M}_1}{\dot{M}_3} = \omega \qquad [3]$$

$$\Pi_2 = \frac{\dot{M}_{G,1}}{\dot{M}_{G,3}} \qquad [4]$$

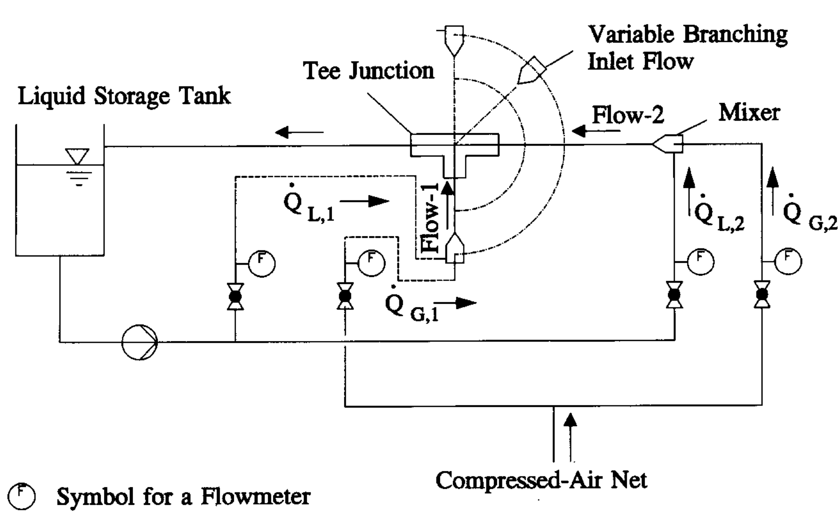


Figure 2. Air–water test loop.

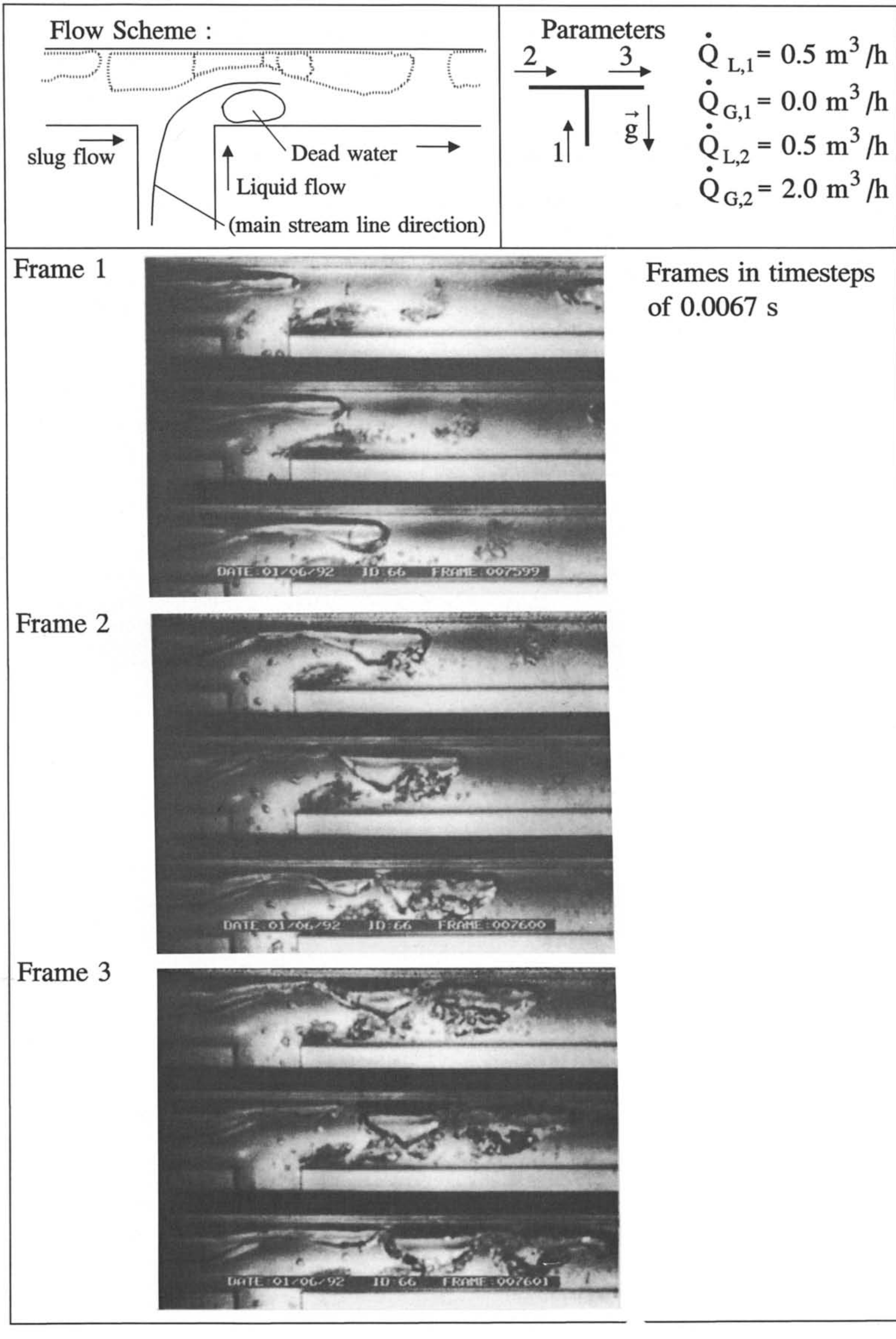


Figure 3. Qualitative air–water experiments. Liquid flow combined with horizontal slug flow.

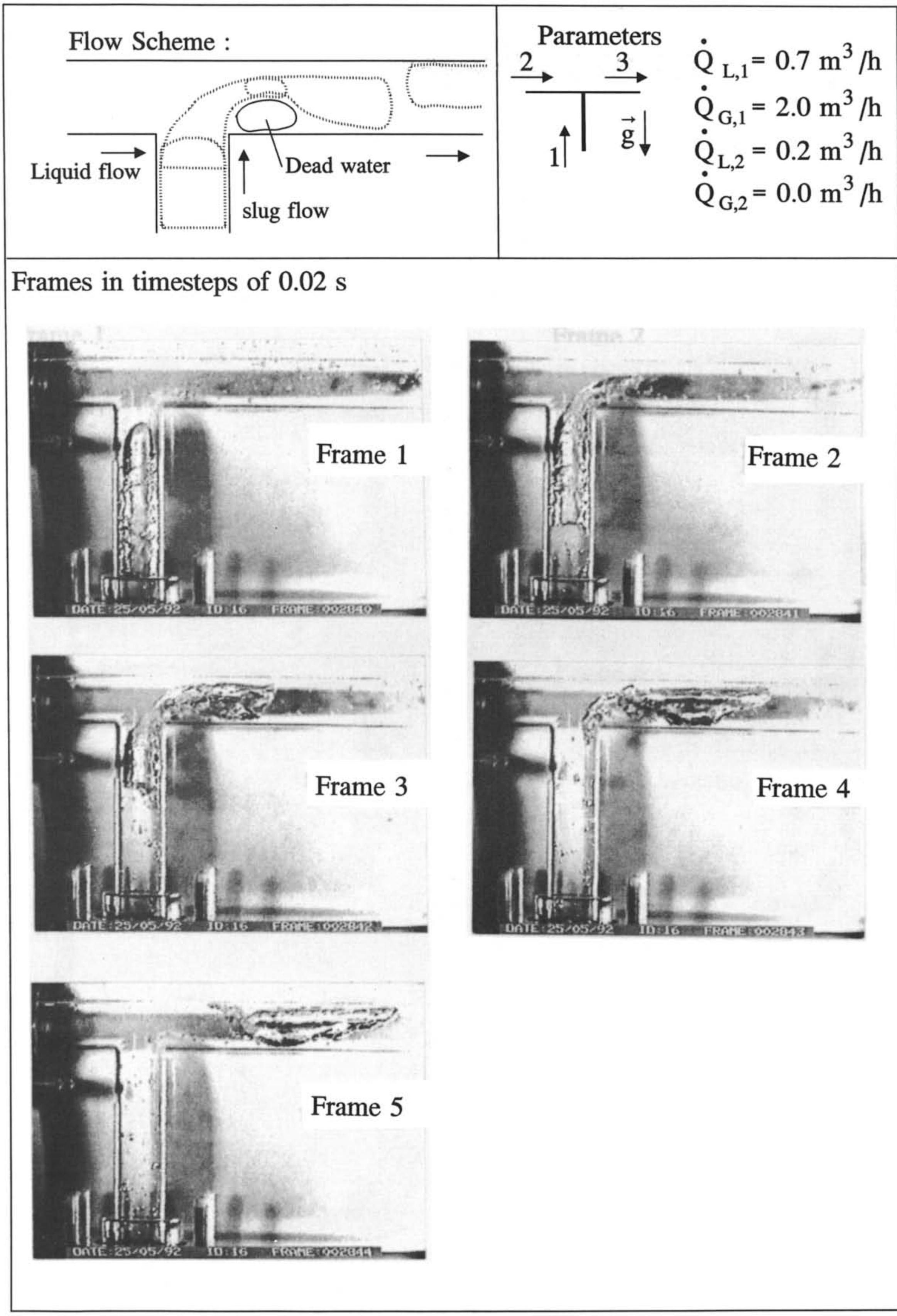


Figure 4. Qualitative air–water experiments. Liquid flow combined with vertical slug flow.

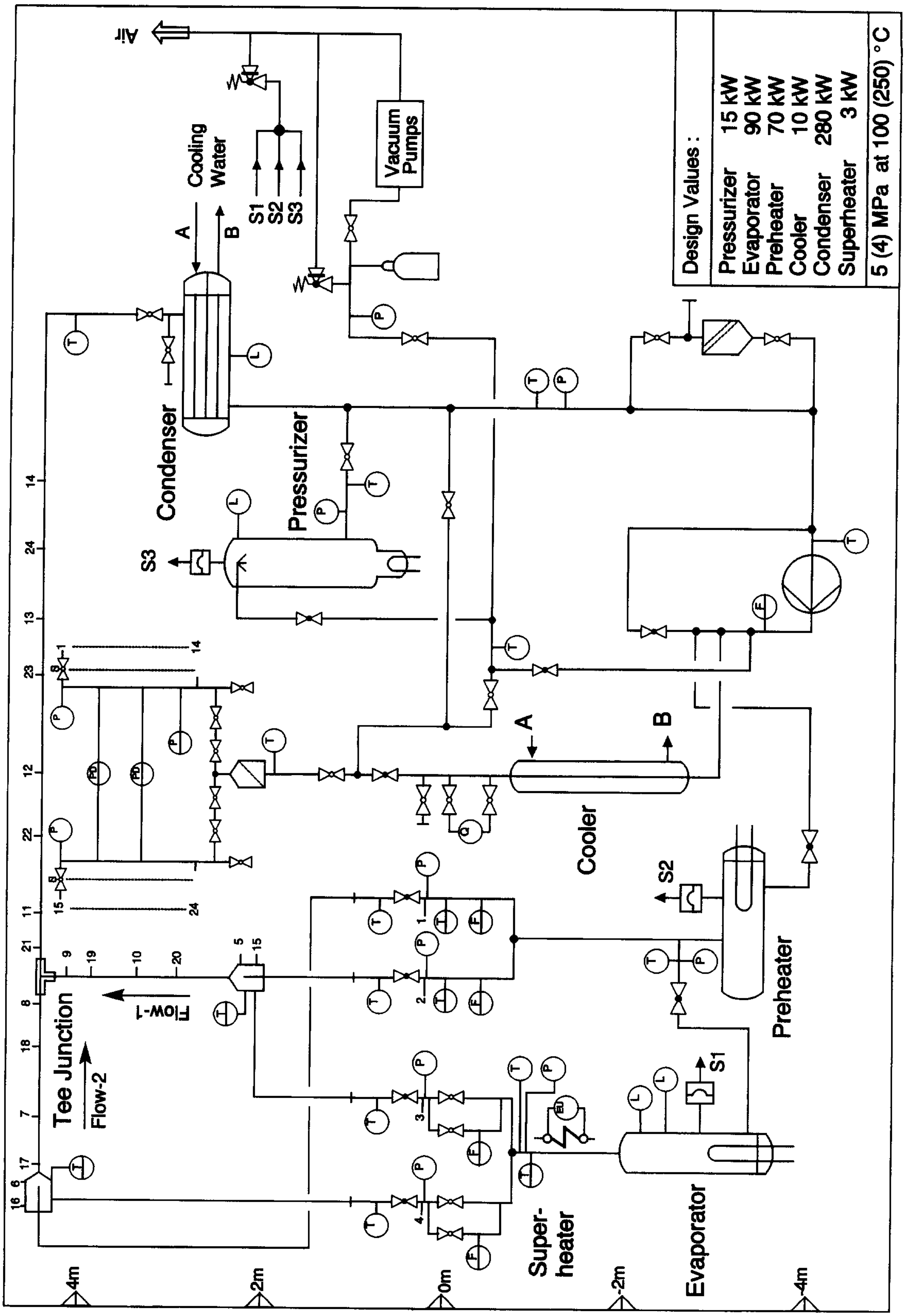
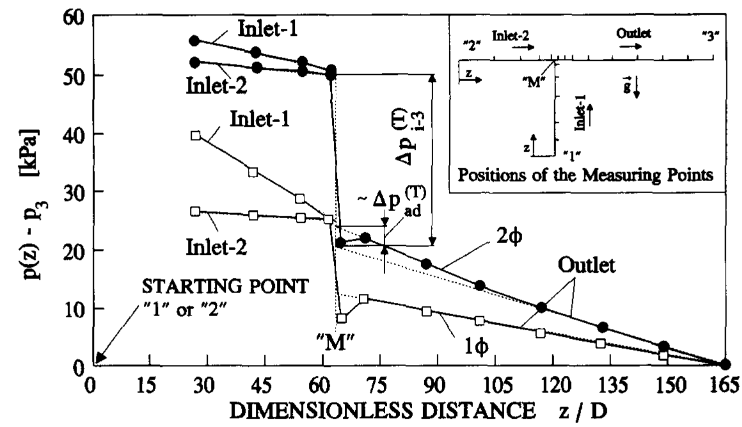


Figure 5. Refrigerant (R12) test loop.



	$\dot M_1$ [kg/s]	$\dot M_3$ [kg/s]	x_1 [-]	x_3 [-]	p_3 [MPa]
Single-phase flow (1ϕ)	1.5	3.0	0.0	0.0	0.8
Two-phase flow (2ϕ)	0.6	1.2	0.3	0.3	0.8

Figure 6. Measured pressure gradients in the tee of the refrigerant test loop for single (1ϕ)- and two-phase (2ϕ) flow.

$$\Pi_3 = \frac{\dot M_{G,3}}{\dot M_3} = X_3 \qquad [5]$$

Here $\dot M_1$, $\dot M_3$ = total mass flow rate for inlet-1, outlet and $\dot M_{G,1}$, $\dot M_{G,3}$ = gas mass flow rate, respectively.

For each experiment the desired flows (annular or churn flow at inlet-1 and slug or annular flow at inlet-2) were passed through the tee test section. The flow pattern maps of Hewitt & Roberts (1969) and of Taitel & Dukler (1976) helped to define the flow pattern. Figure 8 shows, as example, the superficial gas (V_{GS}) and liquid (V_{LS}) velocities of the inlet-1 flows of the experiments, drawn in a flow pattern map.

Figure 9 shows (only leg inlet-1 is shown) that the pressure change, respectively the Euler number (where $\rho_{h,3}$ is the average density and $v^2_{h,3}$ the square of the average velocity at position "3" of the outlet), increases with the mass flow ratio $\dot M_1/\dot M_2$. For mass flow ratios near zero term $\Delta p^{(p)}_{1\text{-}3}$ dominates. In total approximately a thousand experimental cases were run and the measured values

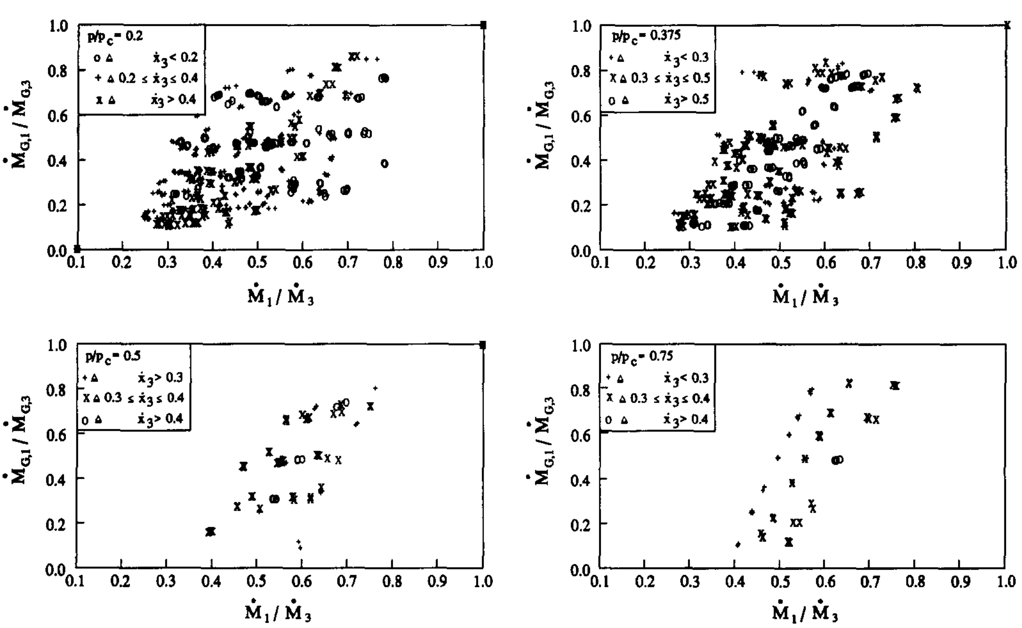


Figure 7. Parameter ranges for the flow ratios and reduced pressure values used in the experiments.

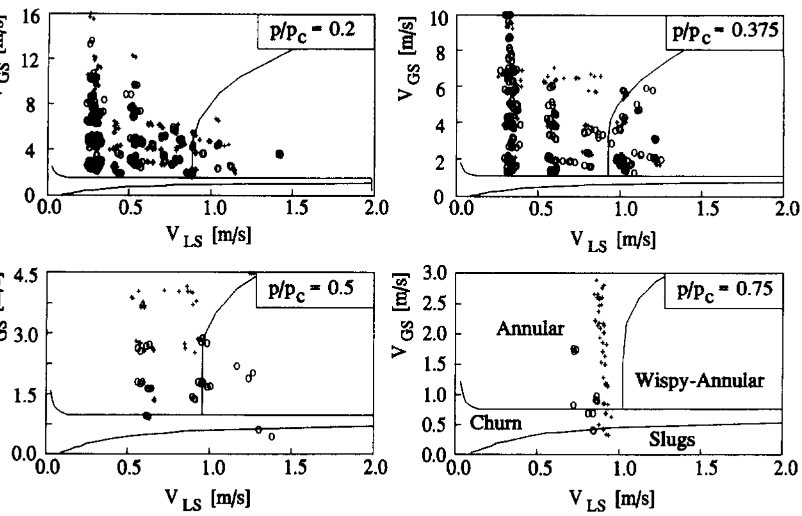


Figure 8. Experimental data points for inlet-1 shown with the flow pattern map of Hewitt and Roberts. Inlet-2 flow was slug flow and annular flow.

for term $\Delta p_{i\text{-}3}^{(T)}$ were between about 0.5 and 3.5 MPa. Figure 10 shows the influence of the reduced pressure on the Euler number. With increasing pressure, the measured pressure distributions approach those of the single-phase flows.

3.PRESSURE LOSS CALCULATION MODELS

3.1. Three Basic Methods

Based on known methods three predictive methods for pressure loss calculation in tee junctions with combining conduits were developed. They will be presented as the "loss coefficient model", "contraction coefficient model" and "momentum coefficinet model". In all models the term $\Delta p_{i\text{-}3}^{(T)}$ is divided into two parts.

$$\Delta p_{i\text{-}3}^{(T)} = \Delta p_{i\text{-}3b}^{(T)} + \Delta p_{3b\text{-}3}^{(T)}; \quad \Delta p_{3b\text{-}3}^{(T)} \equiv \Delta p_{ad}^{(T)} \qquad [6]$$

$\Delta p_{i\text{-}3b}^{(T)}$ describes the pressure change from the inlet to a position 3b in the outlet, where the two inlet flows are mixed and the flow fills the total cross-section at first. At position 3b the flow is more homogenized than at position 3, where the flow corresponds to the fully developed pipe flow. The additional pressure change $\Delta p_{ad}^{(T)}$ shall be attributed to the process of development of the final flow pattern between position 3b and 3. This term will be discussed in section 3.2.2.

For better understanding of the following sub-sections 3.1.1, 3.1.2 and 3.1.3, where the principles of the models are defined, notice that the different coefficients which go along with each model are

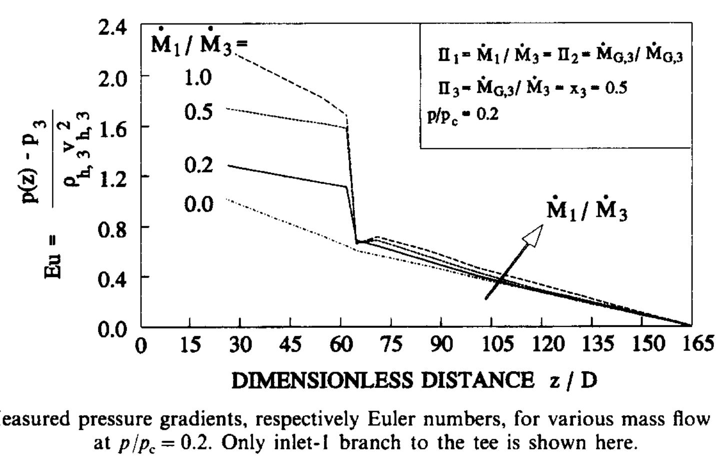


Figure 9. Measured pressure gradients, respectively Euler numbers, for various mass flow ratios $\dot{M}_1/\dot{M}_3$ at $p/p_c = 0.2$. Only inlet-1 branch to the tee is shown here.

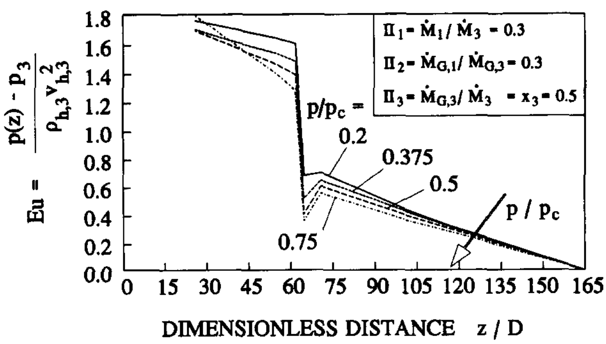


Figure 10. Measured pressure gradients, respectively Euler numbers, for various p/p_c values with $\dot{M}_1/\dot{M}_3 = 0.3$. Inlet-1 branch to the tee is shown here.

always traced back to a single-phase correction coefficient as a function of a parameter ω (definable as ratios of different physical parameters). The idea is to use this same function in the models [i.e. $k_{2\phi} = k_{1\phi}(\omega)$, where it suffices that 2ϕ and 1ϕ denote two-phase and single-phase flow]. The ω parameter was defined in different ways (see [33]–[39]) and for each case considered calculated according to the chosen two-phase flow model.

3.1.1. Loss coefficient model

The assumed flow scheme of this model is shown in figure 11. The pressure change $\Delta p^{(t)}_{i\text{-}3b}$ is divided into a reversible and an irreversible component.

$$\Delta p^{(T)}_{i\text{-}3b} = (\Delta p^{(T)}_{i\text{-}3b})_{rev} + (\Delta p^{(T)}_{i\text{-}3b})_{irrev} \qquad [7]$$

To predict the reversible pressure change fictitous stream tubes were assumed from the inlet to position 3b and the energy balances are solved under the following assumptions:

- —quasi steady-state flow
- —No heat exchange and/or production
- —$A_{3b} = A_{3b,1} + A_{3b,2} = A_3$
- —fluid parameters are constant
- —no phase change
- —no influence of the gravitational forces
- —pressure is constant in each cross-section
- —forces on the contour of a stream tube only affect the exchange of mechanical energy (L_i) between the two stream tubes

It can be written for the reversible term:

$$p_i \int_{A_i} v \, dA - p_{3b} \int_{A_{3b,i}} v \, dA = \int_{A_{3b,i}} \frac{\rho v^3}{2} dA - \int_{A_i} \frac{\rho v^3}{2} dA + L_i \qquad [8]$$

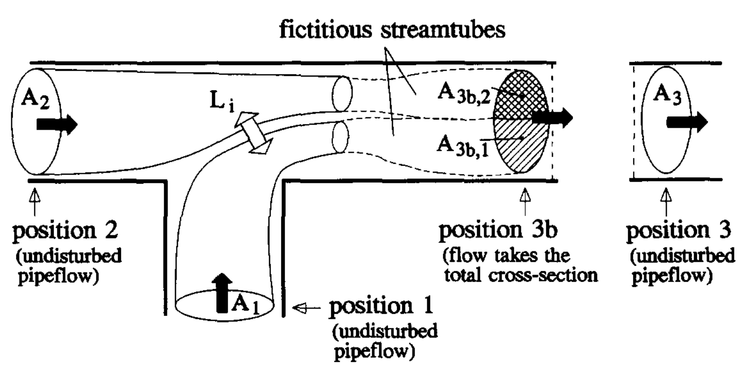


Figure 11. Flow scheme for the "loss coefficient model".

where ρ = flow density, v = flow velocity, $L_1 = -L_2$, A_i = inlet cross-section of branch i and $A_{3b,i}$ = cross-section of the stream tube of flow i at position 3b. These are fictitious cross-sections, since the two inlet flows are already mixed at that location as the qualitative experiments have shown. For simpler handling of the integrals a streamline correction factor is introduced, which is related to the homogeneous flow model. For the expression $(\rho^m v^n)$ and the ith-branch of the tee it is defined as

$$F_{\langle\rho^m v^n\rangle i} = \frac{\dfrac{1}{A_i}\dfrac{1}{\Delta\tau}\displaystyle\int_{A_i}\int_0^{\Delta\tau}\rho^m v^n \,\mathrm{d}t\,\mathrm{d}A}{\rho_{\mathrm{h},i}^m v_{\mathrm{h},i}^n} = \frac{\overline{\langle\rho^m v^n\rangle_i}}{\rho_{\mathrm{h},i}^m v_{\mathrm{h},i}^n} \qquad [9]$$

where m, n = dummy exponents, suffix h indicates homogeneous model assumptions and $\Delta\tau$ = time interval. The streamline correction factor at position 3b is then defined as

$$F_{\langle\rho^m v^n\rangle 3\mathrm{b},i} = \frac{\dfrac{1}{\Delta\tau}\displaystyle\int_{A_{3\mathrm{b}}}\int_0^{\Delta\tau}\left[R\rho_{\mathrm{G}}^m v_{\mathrm{G}}^n \frac{\dot{M}_i x_i}{\dot{M}_3 x_3} + (1-R)\rho_{\mathrm{L}}^m v_{\mathrm{L}}^n \frac{\dot{M}_i(1-x_i)}{\dot{M}_3(1-x_3)}\right]\mathrm{d}t\,\mathrm{d}A}{\rho_{\mathrm{h},3}^m v_{\mathrm{h},3}^n \dfrac{\dot{M}_i}{\dot{M}_3} A_3} \qquad [10]$$

where R is a gas state density function and x is the flow quality. Since the expression in the denominator is based on the homogeneous model assumptions the suffix 3 at the variable ρ, v and A is exchangeable with suffix 3b.

These streamline correction factors become per definition one for a homogeneous flow model. Using [9] and [10] the reversible term of the pressure change in [7] can be expressed as:

$$(\Delta p_{i\text{-}3\mathrm{b}}^{(\mathrm{T})})_{\mathrm{rev}} = (p_i - p_{3\mathrm{b}})_{\mathrm{rev}} = \rho_{\mathrm{h},i}\frac{v_{\mathrm{h},3}^2}{2}F_{\langle\rho v^3\rangle 3,\mathrm{b},i} - \frac{\rho_{\mathrm{h},i}v_{\mathrm{h},i}^2}{2}F_{\langle\rho v^3\rangle i} + \frac{L_i\rho_{\mathrm{h},i}}{\dot{M}_i} \qquad [11]$$

The irreversible term of [7] can be written:

$$(\Delta p_{i\text{-}3\mathrm{b}}^{(\mathrm{T})})_{\mathrm{irrev}} = \left(\frac{\rho_{\mathrm{h},i}v_{\mathrm{h},3}^2}{2}F_{\langle\rho v^3\rangle 3\mathrm{b},i} + \frac{L_i\rho_{\mathrm{h},i}}{\dot{M}_i}\right)k_{\mathrm{L},i\text{-}3,2\phi} \qquad [12]$$

This equation is similar to the equation for turbulent incompressible single-phase flow except the L_i term. According to the chosen procedure, keeping the functional relationship between the two-phase and the single-phase loss coefficients, we search for an appropriate relation for $k_{\mathrm{L},i\text{-}3,1\phi}$ first. For the single-phase flow loss coefficient $k_{\mathrm{L},i\text{-}3,1\phi}$ many correlations exist in the literature (e.g. Gardel 1971). Our own single-phase flow measurements, carried out during this investigation, resulted in the following equations:

$$k_{\mathrm{L},1\text{-}3,1\phi} = 2.065\omega^3 - 4.604\omega^2 + 4.764\omega - 0.939 \qquad [13]$$

$$k_{\mathrm{L},2\text{-}3,1\phi} = 0.871\omega^3 - 2.190\omega^2 + 1.896\omega + 0.065 \qquad [14]$$

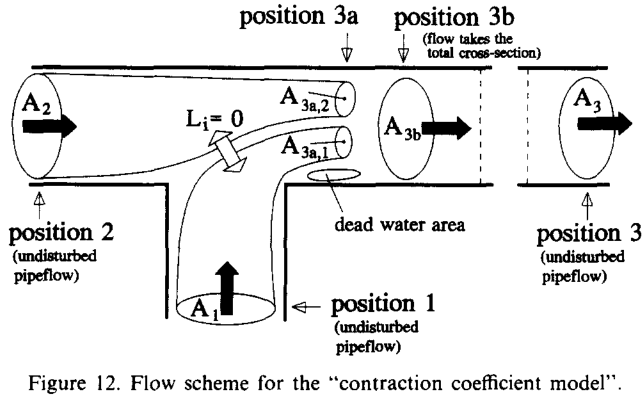


Figure 12. Flow scheme for the "contraction coefficient model".

3.1.2. *Contraction coefficient model*

The assumed flow scheme of this model is shown in figure 12. It assumes a deadwater area just after the tee junction in the outlet according to the indication of the qualitative air–water experiments and to the investigations of Schmid (1977) for single-phase flow. The flow through the tee junction is divided into two regions, one before the maximal contraction and one after. Further it is defined, that both inlet flows reach their maximal contraction at the plane position 3a in the outlet. The cross-section of each flow from branch i at 3a is then defined by a contraction coefficient

$$A_{3a,i} = A_3 \cdot k_{C,i-3a} \qquad [15]$$

and we write:

$$\Delta p^{(T)}_{i\text{-}3b} = \Delta p^{(T)}_{i\text{-}3a} + \Delta p^{(T)}_{3a\text{-}3b} \qquad [16]$$

Up to the contraction-plane the flow is assumed to be non-dissipative and the pressure change $\Delta p^{(T)}_{i\text{-}3a}$ is estimated by an energy balance. With the same assumptions as described in section 3.1.1 it can then be written:

$$\Delta p^{(T)}_{i\text{-}3a} = \frac{\rho_{h,i} v^2_{h,i}}{2}\left(\frac{F_{\langle \rho v^3\rangle 3a,i}}{k^2_{C,i\text{-}3a}} - F_{\langle \rho v^3\rangle i}\right) + \frac{L_i \rho_{h,i}}{\dot{M}_i} \qquad [17]$$

After the contraction-plane the flow is assumed to be dissipative. The pressure change term $\Delta p^{(T)}_{3a-3b}$ is estimated by a momentum balance in the flow direction. Using the following assumptions

—quasi steady-state flow
—fluid parameters are constant
—no phase change
—no influence of the gravitation
—pressure in each cross-section is constant
—shear stress at the wall of the pipe is equal to that of the undisturbed pipe flow it can be written:

$$\int_{A_{3a}} p\,\mathrm{d}A - \int_{A_{3b}} p\,\mathrm{d}A = \int_{A_{3b}} \rho v^2\,\mathrm{d}A - \int_{A_{3a,1}} \rho v^2\,\mathrm{d}A - \int_{A_{3a,2}} \rho v^2\,\mathrm{d}A \qquad [18]$$

or with the continuity assumption for each stream tube "1–3a", "2–3a", "3b–3" and with $A_i = A_3$ it follows:

$$\Delta p^{(T)}_{3a\text{-}3b} = \rho_{h,3} v^2_{h,3} F_{\langle \rho v^2\rangle 3b} - \frac{\rho_{h,1} v^2_{h,1} F_{\langle \rho v^2\rangle 3a,1}}{k_{C,1\text{-}3a}} - \frac{\rho_{h,2} v^2_{h,2} F_{\langle \rho v^2\rangle 3a,2}}{k_{C,2\text{-}3a}} \qquad [19]$$

For defining the contraction coefficients $k_{C,i-3a}$ as functions of the known loss coefficients $k_{L,i-3,1\phi}$, one starts from a limiting case. It was assumed that at flow quality one or zero the model (with $Li = 0$, $\Delta p^{(T)}_{ad} = 0$ and $F\langle\rangle = 1$) yields the same $\Delta p^{(T)}_{i\text{-}3}$-value as a single-phase model for turbulent incompressible flow. Equating both Δp-equations and replacing $k_{L,i\text{-}3,1\phi}$ in the single-phase model by [13] and [14], the following solutions yield via an iteration method:

$$k_{C,1\text{-}3a} = 0.159\omega^3 - 0.418\omega^2 + 0.719\omega + 0.018 \qquad [20]$$

$$k_{C,2\text{-}3a} = -0.583\omega^3 + 1.277\omega^2 - 1.487\omega^2 - 1.487\omega + 0.783 \qquad [21]$$

These approximation solutions are valid for a wide ω range but the endpoints zero and one, where no definite solutions exist.

3.1.3. *Momentum coefficient model*

The assumed flow scheme of this model is shown in figure 13. The basis of this model is a momentum balance in the flow direction, and therefore it can only be used for estimating the pressure change $\Delta p^{(T)}_{2\text{-}3b}$. With the same assumptions as described in section 3.1.2 and the additional assumption that the velocity of inlet-1 flow is normal to the horizontal flow direction, it can be written:

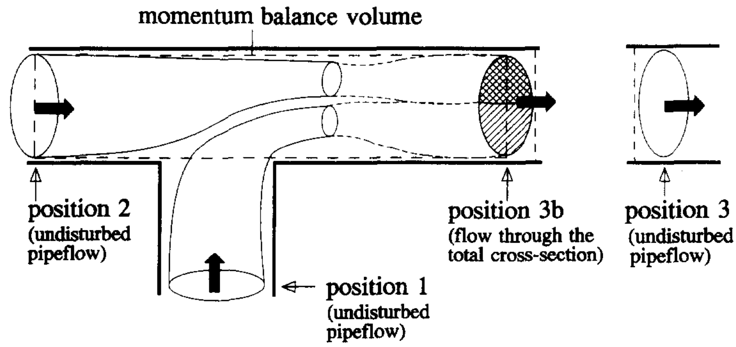


Figure 13. Flow scheme for the "momentum coefficient model".

$$(\Delta p^{(T)}_{2\text{-}3b})_{\text{ideal}} = \frac{1}{A_3}\left(\int_{A_{3b}} \rho v^2 \, dA - \int_{A_2} \rho v^2 \, dA\right) \quad \text{with } A_3 = A_{3b} = A_2 \qquad [22]$$

Using the streamline correction factors and the momentum correction coefficients, yields:

$$\Delta p^{(T)}_{2\text{-}3b} = (\rho_{h,3} v^2_{h,3} F_{\langle \rho v^2 \rangle 3b} - \rho_{h,2} v^2_{h,2} F_{\langle \rho v^2 \rangle 2}) k_{M,2\text{-}3b,2\phi} \qquad [23]$$

For defining coefficient $k_{M,2-3b,2\phi}$ as a function of the parameter ω again, like in section 3.1.2, a limiting case was chosen, so one could equate the corresponding $\Delta p^{(T)}_{2\text{-}3b}$ equations and derive hereof the following relationship:

$$k_{M,2\text{-}3b,1\phi} = \frac{1 + k_{L,2\text{-}3,1\phi} - (1-\omega)^2}{1 - (1-\omega)^2} \qquad [24]$$

3.2. *The Open Parameters of the Models*

The basic models described in section 3.1 were derived using the following "open parameters", which will now be discussed:

—streamline correction factors
—additional pressure change
—correction coefficients
—L-term (only in the loss- and contraction-coefficient model)

3.2.1. *Streamline correction factors*

Equations [9] and [10] for the streamline correction factors were evaluated under the assumption of three two-phase flow models:

—the homogeneous model
—the separated flow model
—a variable density model.

In the homogeneous model the streamline correction factors equal unity per definition.

The so-called separated flow model, i.e. mixture model, considers two distinct average phase velocities $\langle v_G \rangle$ and $\langle v_L \rangle$, and the velocity ratio

$$S = \frac{\langle v_G \rangle}{\langle v_L \rangle}$$

is a quantity often used. This quantity is related to the flow quality x and the void fraction $\langle \epsilon \rangle$ by the following relation:

$$\langle \epsilon \rangle = \frac{1}{1 + S\dfrac{\rho_G}{\rho_L}\dfrac{1-x}{x}} \qquad [25]$$

Then it can be written:

$$\frac{1}{\Delta\tau}\frac{1}{A}\int_0^{\Delta\tau}\int_A \rho^m v^n\,\mathrm{d}A\,\mathrm{d}t = \left(\frac{\dot{M}}{A}\right)^n [x^n \rho_G^{m-n}\langle\epsilon\rangle^{1-n} + (1-x)^n \rho_L^{m-n}(1-\langle\epsilon\rangle)^{1-n}] \quad [26]$$

Here, equations from Hughmark (1962), Melber (1989), Nabizadeh-Arghi (1977), Premoli *et al.* (1971) and Rouhani (1969) have been used to describe the flow quality. Because of the qualitative experiments at the air–water test facility a homogenization at positions 3a and 3b can be expected, i.e. one could assume for the velocity ratio:

$$S_{3a,1} = S_{3a,2} = S_{3b} = 1 \quad [27]$$

Since this is not proven, it was also assumed alternatively that the velocity ratios at positions 3a and 3b equal the ratios at the positions with fully developed flows.

$$S_{3a,1} = S_1; \quad S_{3a,2} = S_2; \quad S_{3b} = S_3 \quad [28]$$

Further a variable density model described by Melber (1989) has been used. This model is only valid at the positions with undisturbed pipe flow, i.e. at positions 1, 2 and 3. (At positions 3a and 3b the slip model was being used.) This model describes the concentration of vapour bubbles across the channel via a parabolic equation. The velocity profile is estimated via the derivation of the pressure loss transferring Prandtl's mixing length theory to the two-phase flow situation.

3.2.2. Additional pressure change term

Two different assumptions have been tested:

(1) Becasue it was not clear from the beginning, if the additional pressure change term has a significant influence on the term $\Delta p_{i-3}^{(T)}$ it has also been set to zero, i.e.:

$$\Delta p_{ad}^{(T)} = 0 \quad [29]$$

(2) The flow at position 3b is more homogenized than at position 3, and the two phases separate more and more downstream from 3b. This separation is due to the wall friction which is higher than for the developed pipe flow. The difference between these two terms can be written as a pressure change that is set to correspond to the difference of the kinetic energy flow at positions 3b and 3.

This yields the friction term:

$$\Delta p_f^{(T)} = \frac{\rho_{h,3} v_{h,3}^2}{2}(F_{\langle\rho v^3\rangle 3b} - F_{\langle\rho v^3\rangle 3}) \quad [30]$$

The additional pressure change can then be estimated by a momentum balance in the flow direction.

$$\Delta p_{ad}^{(T)} \equiv \Delta p_{3b-3}^{(T)} = \rho_{h,3} v_{h,3}^2 (F_{\langle\rho v^2\rangle 3} - F_{\langle\rho v^2\rangle 3b}) + \Delta p_f^{(T)} \quad [31]$$

3.2.3. Correction coefficients

Each of the basic models includes a correction coefficient. These coefficients can be estimated, without problem, for turbulent incompressible single-phase conditions. The single-phase correction factor correlations are normally written as a function of the mass flow ratio ω. It is now assumed that the same correlations describing the correction factor for single-phase flow can be overtaken for two-phase flow situations at similar boundary conditions, i.e.:

$$k_{j,i\text{-}3,2\phi} = k_{j,1\text{-}3,1\phi}(\omega) \quad [32]$$

Index j = L, C, M stands for the three models.

Here attention must be given to the variable ω ([33]–[39]) which has to be calculated according to the chosen two-phase model and normally differs from the values for single-phase flow. For turbulent incompressible single-phase flow with the assumption that the area-averaged flow density is independent of the location, i.e. $\rho_1 = \rho_2 = \rho_3$, and with $A_1 = A_2 = A_3$ the following relations for the parameter ω can be derived:

$$\omega = \frac{\dot{M}_1}{\dot{M}_3} = \frac{\rho_1 v_1 A_1}{\rho_3 v_3 A_3} \tag{33}$$

$$\omega = \frac{Q_1}{Q_3} = \frac{v_1 A_1}{v_3 A_3} \tag{34}$$

$$\omega = \sqrt{\frac{\dot{I}_1}{\dot{I}_3}} = \sqrt{\frac{\rho_1 v_1^2}{\rho_3 v_3^2}} \tag{35}$$

$$\omega = \sqrt[3]{\frac{\dot{E}_{\text{kin},1}}{\dot{E}_{\text{kin},3}}} = \sqrt{\frac{\rho_1 v_1^3}{\rho_3 v_3^3}} \tag{36}$$

$$\omega = \frac{1}{1 + \dfrac{Q_2}{Q_1}} \tag{37}$$

$$\omega = \frac{1}{1 + \sqrt{\dfrac{\dot{I}_2}{\dot{I}_1}}} \tag{38}$$

$$\omega = \frac{1}{1 + \sqrt[3]{\dfrac{\dot{E}_{\text{kin},2}}{\dot{E}_{\text{kin},1}}}} \tag{39}$$

While the mass flow and the volume flow ratios stay unchanged for the two-phase flow, this is no longer valid for the momentum and the kinetic energy fluxes. This is the reason why the ratios in [35], [36] and [38], [39] have been alternatively calculated for the study with the two-phase flow models described in section 3.2.1.

3.2.4. L-term

The "loss-" and the "contraction coefficient" models include an L-term that accounts for the mechanical energy exchange between the surfaces of the two stream tubes. Two assumptions were made to see its influence on the pressure change $\Delta p^{(T)}_{i-3}$.

Assumption 1 (used for both models):

$$L = 0 \tag{40}$$

Along with the "loss coefficient model" it was also assumed $L \neq 0$. Here the total kinetic energy flux for each flow i, calculated with the homogeneous model, is assumed to be

$$E_{\text{kin},3\text{b},i} = \int_{A_{2\text{b},i}} \left(\frac{\rho v^3}{2}\right)_{\text{h}} \text{d}A = \frac{\dot{M}_i v_{\text{h},i}^2}{2} \tag{41}$$

The L-term can be expressed in the following simple way:

It is proportional to the difference between the relative velocity and mass flow ratio of the inlet flow to the outlet

$$L_i \sim \left(\frac{v_{\text{h},i}}{v_{\text{h},3}} - \frac{\dot{M}_i}{\dot{M}_3}\right) \Big/ \frac{\dot{M}_i}{\dot{M}_3} \tag{42}$$

and it is proportional to the kinetic energy flux at the outlet

$$L_i \sim \frac{\dot{M}_i v_{\text{h},3}^2}{2} \tag{43}$$

Combining [42] and [43] leads to

Assumption 2:

$$L_i = \frac{\dot{M}_i v_{\text{h},3}^2}{2}\left(\frac{\dot{M}_3}{\dot{M}_i}\frac{v_{\text{h},i}}{v_{\text{h},3}} - 1\right) \tag{44}$$

The calculations can be simplified if the following expression is used:

$$\frac{\rho_{h,i}}{\dot{M}_i}(\dot{E}_{kin,h,3b,i} + L_i) = \frac{\rho_{h,i}}{\dot{M}_i}\left(\frac{\dot{M}_i v_{h,3}^2}{2} + \frac{\dot{M}_i v_{h,3}^2}{2}\left(\frac{\rho_{h,3}}{\rho_{h,i}} - 1\right)\right) = \frac{\rho_{h,3} v_{h,3}^2}{2} \qquad [45]$$

4. RESULTS

With each basic model described in section 3.1 and with any possible combination to determine the open parameters, defined in section 3.2, the pressure change term $\Delta p_{i-3}^{(T)}$ has been calculated and compared with measured data. Almost more than 500 combinations of calculations were executed and checked. The relative differences between measured and calculated values for the term $\Delta p_{i-3}^{(T)}$ have been plotted as a function of the parameters $\Pi 1$, $\Pi 2$ and $\Pi 3$ (see [3]–[5]). The only way to emphasize tendencies was by plotting the relative differences between the measured and calculated pressure drops as shown in figure 14 (for more details see Schmidt 1993).

It was possible to reproduce the measured pressure changes by using each of the basic models. The best agreement between measurement and calculation could be achieved by using the same set of values for the "open parameters" in each model. The additional pressure change term must be considered according to [31]. For this purpose the slip ratio at position 3b has to be set to a value of one and at position 3 to a value corresponding to the undisturbed pipe flow. Otherwise, the calculated pressure changes are too low.

The correction coefficients mainly depend on the definition of the variable ω. The best agreement was found for ω being a function of the momentum flux ratio. On the other hand, the difference was small if the ratio according to [35] or [38] was taken. Nevertheless, it makes a big difference according to which two-phase flow model the momentum flux ratio is calculated. The best results were obtained with the homogeneous model using [38]. The study has also shown, especially for calculating the pressure change $\Delta p_{i-3}^{(T)}$ with the "loss coefficient model" and the "contraction coefficient model", that ω should be derived from [38]. For the calculation of $\Delta p_{2-3}^{(T)}$ with the "momentum coefficient model", the way to determine ω is less important.

The L-term should be set to zero for the "contraction coefficient model", but for the "loss coefficient model" it should be evaluated according to [44] to fit the experimental data best.

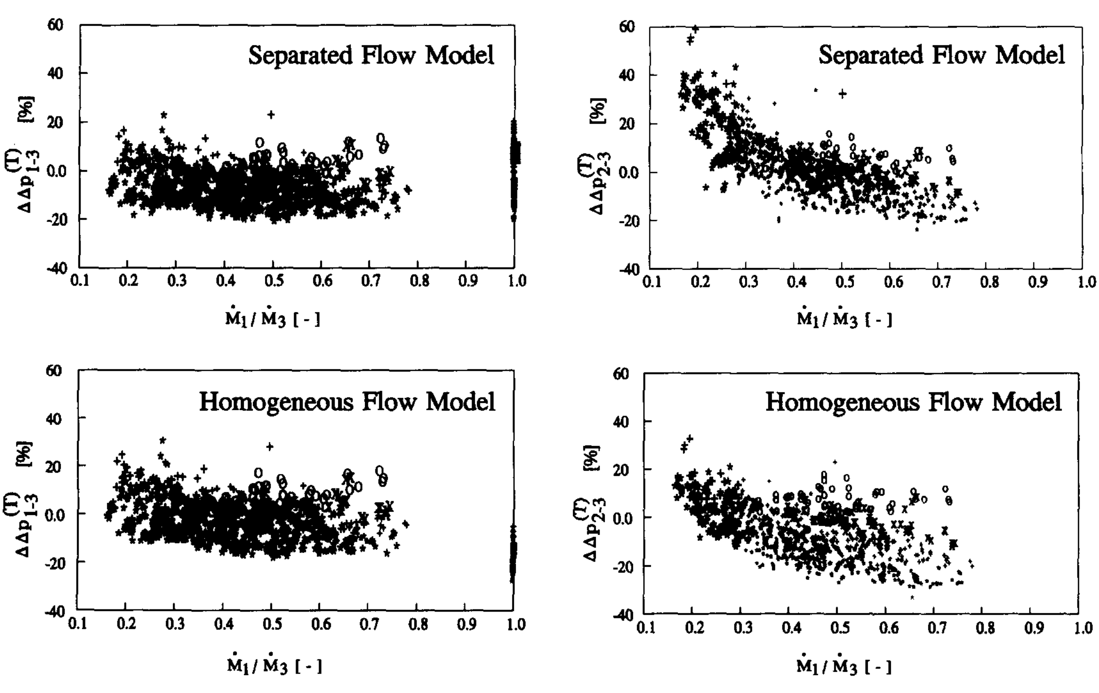


Figure 14. Relative errors between measured and calculated pressure drops using the separated flow model and homogeneous model: $\Delta\Delta p_{i-3}^{(T)} = 100 \times (\Delta p_{i-3,calc.}^{(T)} - \Delta p_{i-3,meas.}^{(T)})/\Delta p_{i-3,meas.}^{(T)}$

The various feasible ways to estimate the cross-section averaged product $\overline{\langle \rho^m v^n \rangle}$ by using different two-phase flow models were reduced to an estimate of the streamline correction factors.

The best agreement is obtained for $\Delta p_{1-3}^{\{T\}}$, the pressure change from the side inlet branch to the outlet, if the slip model is used, and the slip ratios at positions 3a and 3b are set to unity. On the other hand $\Delta p_{2-3}^{\{T\}}$, the pressure change from the horizontal inlet branch to the outlet, could be reproduced best with the homogeneous model.

With the variable density model very similar results have been reached as with the slip model (differences below 3%), independent from the basic model and the assumptions for the open parameters. The variable density model is very complicated to apply here.

With respect to the way the flow quality was described (see section 3.2.1), no significant influence could be found.

5. CONCLUSIONS

The objective of the project was to provide a practicable and sufficiently accurate method for calculating the pressure drop at a tee junction with combining conduits. Three basic models, termed the "loss coefficient model", "contraction coefficient model" and "momentum coefficient model" were derived. Special parameters, here termed "open parameters", in the models had to be determined first. This was done assuming plausible physical models. Subsequently, many pressure change calculations, with all three models, and alternating descriptions of the open parameters were performed for corresponding experiments. The experiments covered the flow ratio range $0.15 \leqslant \dot{M}_1/\dot{M}_3 \leqslant 0.75$ and the reduced pressure range $0.2 \leqslant p/p_c \leqslant 0.75$. The fluid was R12. Comparison between the measured and predicted pressure change values allows recommendation of the formula given in the appendix.

The two different expressions for the streamline correction factors at positions 1 and 2 are the result of the semi-empirical approach. The parameters which have to be fitted are almost equal (except the exchanged energy L_i-term) for all basic models. This might be an indication for good agreement between the chosen models and the physics.

It seems that an additional pressure change term has to be considered. However, it is best to use the momentum flux ratio, calculated with the homogeneous model, for determining the correction coefficients.

REFERENCES

GARDEL, A. & RECHSTEINER, G. F. 1971 Les pertes de charge dans les branchements en Té des conduites de section circulaire. Société du Bulletin de la Swiss Romande, Publication 118, "Ecole polytechnique fédérale de Lausanne", Lausanne.

HEWITT, G. F. & ROBERTS, D. N. 1969 Studies of two-phase flow patterns by simultaneous X-ray and flash photography. AERE-M2159.

HUGHMARK, G. A. 1962 Hold-up in gas–liquid flow. *Chem. Engng Prog.* **58,** 62–65.

MAYINGER, F. 1982 *Strömung und Wärmeübergang in Gas-Flüssigkeits-Gemischen.* Springer, Vienna.

MELBER, A. 1989 Experimentelle und theoretische Untersuchungen über den Druckabfall von Zweiphasenströmungen in beliebig geneigten Rohren. Dissertation Technische Hochschule Darmstadt, Germany.

NABIZADEH-ARAGHI, H. 1977 Modellgesetze und parameteruntersuchungen für den Volumetrischen Dampfgehalt in einer Zweiphasenströmung. Dissertation Universität Hannover, Germany.

PREMOLI, A., DI FRANCESCO, D. & PRINA, A. 1971 Una correlazione adimensionale per la determinazione della densità de miscele bitasiche. Paper presented at the *XXV Congresso Nazionale ATI, September 1979,* Trieste. *Termotecnica* **25,** 17–25.

ROUHANI, S. Z. 1969 Modified correlations for void and two-phase pressure drop. AB Atomenergie, AE-RTV-841.

SABA, N. & LAHEY JR, R. T. 1984 The analysis of phase separation phenomena in branching conduits. *Int. J. Multiphase Flow* **10,** 1–20.

SCHMID, P. 1977 Beitrag zur experimentellen Untersuchung strömungs-mechanischer Energieverluste in Rohrvereinigung. Dissertation, Technische Universität München, Germany.

SEEGER, W. 1985 Untersuchungen zum Druckabfall und zur Massenstromverteilung von Zweiphasenströmungen in rechtwinkligen Rohrverzweigungen. KfK 3876 and Dissertation Technische Hochschule Karlsruhe, Germany.

TAITEL, Y. & DUKLER, A. E. 1976 Model for predicting flow regime transitions in horizontal and near horizontal gas–liquid flow. *AIChE Jl* **22**, 47–55.

APPENDIX

Required Parameters

For all basic models the following parameters have to be estimated:

—Streamline correction coefficients at positions 1 and 3

$$F_{\langle \rho v^3 \rangle j} = \frac{\dfrac{\dot{M}_j^3}{A_j^3}\left(\dfrac{x_j^3}{\rho_G^2 \langle \epsilon \rangle_j^2} + \dfrac{(1-x_j)^3}{\rho_L^2 (1-\langle \epsilon \rangle_j)^2}\right)}{\rho_{h,j} V_{h,j}^3} \quad \text{with } j = 1 \text{ and } 3$$

$$F_{\langle \rho v^2 \rangle 3} = \frac{\dfrac{\dot{M}_3^2}{A_3^2}\left(\dfrac{x_3^2}{\rho_G \langle \epsilon \rangle_3} + \dfrac{(1-x_3)^2}{\rho_L (1-\langle \epsilon \rangle_3)}\right)}{\rho_{h,3} v_{h,3}^2}$$

—Mass flow ratio

$$\omega = \frac{1}{1 + \sqrt{\dfrac{\rho_{h,2} v_{h,2}^2}{\rho_{h,1} v_{h,1}^2}}}$$

—Loss coefficients

$$k_{L,1\text{-}3,1\phi} = 2.065\omega^3 - 4.604\omega^2 + 4.764\omega - 0.939$$

$$k_{L,2\text{-}3,1\phi} = 0.871\omega^3 - 2.190\omega^2 + 1.896\omega + 0.065$$

—Additional pressure change

$$\Delta p_{ad}^{(T)} = \rho_{h,3} v_{h,3}^2 (F_{\langle \rho v^2 \rangle 3} - 0.5 F_{\langle \rho v^3 \rangle 3} - 0.5)$$

Loss Coefficient Model

—Pressure drop at a tee junction with combining conduits

$$\Delta p_{1\text{-}3}^{(T)} = \frac{\rho_{h,3} v_{h,3}^2}{2}(1 + k_{L,1\text{-}3,1\phi}) - \frac{\rho_{h,1} v_{h,1}}{2} F_{\langle \rho v^3 \rangle,1} + \Delta p_{ad}^{(T)}$$

$$\Delta p_{2\text{-}3}^{(T)} = \frac{\rho_{h,3} v_{h,3}^2}{2}(1 + K_{L,2\text{-}3,1\phi}) - \frac{\rho_{h,1} v_{h,1}^2}{2} + \Delta p_{ad}^{(T)}$$

Contraction Coefficient Model

—Contraction coefficients for $0.05 < \omega < 0.95$

$$k_{C,1\text{-}3} = 0.159\omega^3 - 0.418\omega^2 + 0.719\omega + 0.018$$

$$k_{C,2\text{-}3} = -0.583\omega^3 + 1.277\omega^2 - 1.487\omega + 0.783$$

—Pressure drop at a tee junction with combining conduits

$$\Delta p_{1\text{-}3}^{(T)} = \frac{\rho_{h,1} v_{h,1}^2}{2}\left(\frac{1}{k_{C,1\text{-}3}^2} - F_{\langle \rho v^3 \rangle 1}\right) + \rho_{h,3} v_{h,3}^2 - \frac{\rho_{h,1} v_{h,1}^2}{k_{C,1\text{-}3}} - \frac{\rho_{h,2} v_{h,2}^2}{k_{C,2\text{-}3}} + \Delta p_{ad}^{(T)}$$

$$\Delta p_{2\text{-}3}^{(T)} = \frac{\rho_{h,2} v_{h,2}^2}{2}\left(\frac{1}{k_{c,2\text{-}3}^2} - 1\right) + \rho_{h,3} v_{h,3}^2 - \frac{\rho_{h,1} v_{h,1}^2}{k_{C,1\text{-}3}} - \frac{\rho_{h,2} v_{h,2}^2}{k_{C,2\text{-}3}} + \Delta p_{ad}^{(T)}$$

Momentum Coefficient Model

—Momentum coefficient

$$k_{M,2\text{-}3,2\phi} = \frac{1 + k_{L,2\text{-}3,1\phi} - (1 - \omega)^2}{1 - (1 - \omega)^2}$$

—Pressure drop at a tee junction with combining conduits

$$\Delta p_{2\text{-}3}^{(T)} = (\rho_{h,3} v_{h,3}^2 - \rho_{h,2} v_{h,2}^2) k_{M,2\text{-}3,2\phi} + \Delta p_{ad}^{(T)}$$